SMOOTH BORES IN A TWO-LAYER LIQUID WITH A VELOCITY SHEAR BETWEEN THE LAYERS

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UDC 532.59

The two-dimensional problem of gravity waves on the interface between two nonviscous immiscible liquids with different density, confined between horizontal planes (bottom and top), was studied in [1] on the basis of the second approximation of the theory of shallow water. It was predicted there that there exist stationary periodic waves, solitary internal waves in the form of a "hump" or "hole," and solitary stationary waves of a new type, which are characterized by a smooth monotonic transition of the interface from one constant level to another and are called smooth bores [2]. The parameters of these waves differ from those of other stationary transitions of this type: hydraulic jumps [3, 4], wave jump [5], and monoclinal wave [6], obtained based on the first approximation of shallow-water theory. Smooth bores have not been previously predicted theoretically and have not been observed experimentally, so that it is of great interest to study them experimentally.

Waves in the form of a smooth bore in the undisturbed state of a liquid at rest were first realized and studied in [2]. In this paper the main attention is devoted to the analysis of the behavior of such waves on a shear flow and accompanying reflection from a vertical wall.

The experiments were performed on two experimental setups, diagrams of which are presented in Fig. 1a and b. In the setup *a* the bottom liquid (water with density $\rho_1 = 1$ g/cm^3 and viscosity $\nu_1 = 0.0108$ cm^2/sec) in the undisturbed state moved with a constant velocity and depth. The top liquid (kerosene with $\rho = 0.8$ g/cm^3 and $\nu = 0.0162$ cm^2/sec) remained virtually stationary. Only weak circulatory motion, owing to friction against the interface separating the layers, was observed in it. The total depth of the liquid H = 6 cm, and the working part of the setup is 250 cm long and 18 cm wide.

Waves were generated by the barrier 1, situated at the outlet from the working region of the setup and in the starting state extending above the bottom of the channel to a height b_1 (Fig. 1a). After stationary flow of the bottom liquid with a velocity u_1 and depth h_1 was established in the channel the barrier rapidly moved up to the height b_2 upwards (accompanied by generation of waves of a rise in level) or downwards (accompanied by generation of waves of a lowering of the level). At the same time a disturbance, which can transform into a smooth bore only for a definite value of the depth h_2 uniquely related with b_2, propagates upstream along the flow.

The waves realized in the experiments were compared with the predictions of the theory according to four indications. The first was the fact that with an insignificant deviation from the value of h_2 indicated by the model of [1] the wave generated either is appreciably unsymmetric or it is nonmonotonic: sign-alternating waves, typical for a wave-jump, appear on the interface. The second indication was the independence of the velocity of propagation of the smooth bore v from its amplitude $h_2 - h_1$. Two more indications were obtained by a direct comparison of the experimental and computed data on the velocity and profile of the wave.

In the experiments these data were obtained by two electrical conductivity probes 2 (Fig. 1), placed at a distance x_0 and $x_0 + \Delta x$ from the wave generator. The principle of conversion of the oscillations of the interface into an electric signal is based here on the fact that slightly saline water is a good conductor, while kerosene is a good dielectric. The conversion was linear and satisfied the necessary requirements imposed on the sensitivity and space–time resolution. More detailed information about the probes is given in [7].

The velocity of propagation of the wave v was determined from the transit time of a characteristic point $h_0 = (h_1 + h_2)/2$ on the wave profile over a distance Δx. Repeated measurements under the same conditions showed that the error in the measurement of v did not exceed 3%. Here the function h(x, t) (t is the time) is called the wave profile. For a stationary wave h(x, t) = h(x + vt), which makes it possible to convert the de-

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 45-49, May-June, 1987. Original article submitted April 11, 1986.

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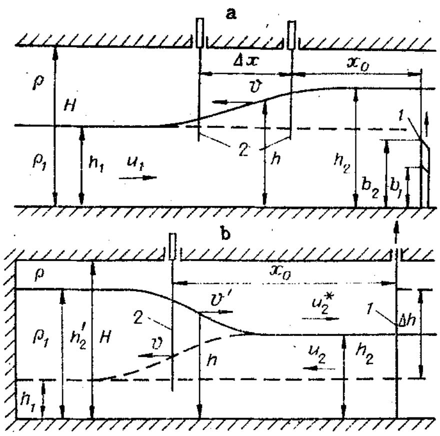

Fig. 1

pendence $h(t)$, obtained with the help of a stationary probe and giving information about its variation at a fixed point x, into the dependence $h(x)$, which would be observed in a coordinate system moving together with the wave. The experimental data presented below are presented previously in this form.

The second setup (see Fig. 1b) was used to study the behavior of the smooth bore as it is reflected from a vertical side wall. It consisted of a channel, closed at the ends, 192 cm long, 20 cm wide, and 6 cm high, separated by an impenetrable barrier 1 into two equal parts. In the undisturbed state both fluids were at rest. To generate waves a level drop $\Delta h = 2(h_2 - h_1)$ was created at the barrier. The barrier was quickly removed, and smooth bores propagated on both sides of it (rising of the level to the left and lowering of the level to the right). A strictly determined depth of the bottom liquid after passage of the smooth bore $h_2 = H/(1 + \sqrt{\lambda})$ was predicted in [1] for a fixed depth H and density ratio $\lambda = \rho/\rho_1$. By varying Δh in order to find waves with different amplitude h_1 must also be varied in a corresponding manner.

Under the conditions of the experiments of [1] we obtain the following expressions for the depth of the bottom liquid behind the bore $y_2 = h_2/H$ and its velocity $V = v/(gH)^{1/2}$ (g is the acceleration of gravity):

$$y_2 = \frac{1 + F_1\sqrt{\frac{\lambda}{\mu}}}{1+\sqrt{\lambda}}, \quad V = \frac{F_1 - \sqrt{\mu}}{1+\sqrt{\lambda}}, \tag{1}$$

where $F_1 = u_1/(gH)^{1/2}$; $\mu = 1 - \lambda$. The profile of the wave in [1] can be found approximately by solving the starting differential equation and is given by the formula

$$y = \frac{y_2 + y_1}{2} + \frac{y_2 - y_1}{2}\,\mathrm{th}\,kx_*. \tag{2}$$

Here $x_* = x/H$; $y = h/H$; $y_1 = h_1/H$; k is a parameter which depends on y_1 and y_2. Computer calculations of the wave profile showed that (2) is virtually identical to its exact expression, so that the calculations carried out based on (1) and (2) are compared with the experimental data.

When both liquids are initially at rest $F_1 = 0$ (Fig. 1b), the depth of the bottom liquid after the passage of the wave and its velocity are determined by the formulas

$$y_2 = 1/(1 + \sqrt{\lambda}), \quad V = -\sqrt{\mu}/(1 + \sqrt{\lambda}).$$

This smooth bore gives rise to a shear flow with velocities $F_2 = [(y_2 - y_1)/y_2]V$, $F_2^* = -[(y_2 - y_1)/(1 - y_2)]V$ of the bottom and top liquids, respectively $[F_2 = u_2/(gH)^{1/2}$, $F_2^* = u_2^*/(gH)^{1/2}]$. The reflected wave now moves along such a shear flow, and if it remains a smooth bore, we obtain $y_2' = 2y_2 - y_1$ and $V' = -V$ for the depth of the liquid behind it $y_2' = h_2'/H$ and its velocity $V' = v'/(gH)^{1/2}$ from [1]. The profile of the reflected wave is described by the expression (2), in which y_2 must be replaced by y_2' and y_1 by y_2 (including also in the expression for k).

We were able to match the conditions for carrying out the calculations and the experimental conditions with respect to all parameters of the problem, except for the viscosity of the liquid and the surface tension at

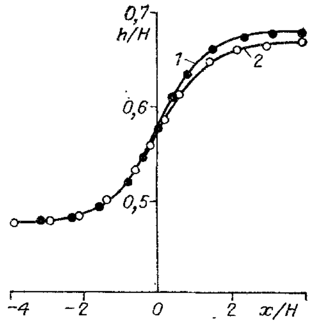

Fig. 2

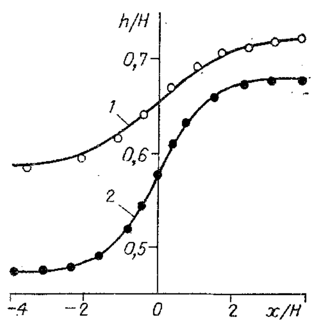

Fig. 3

the interface. The surface tension has virtually no effect on long waves and at the same time plays an important positive role [2, 8]. The viscosity leads to the fact that in the experiments the depth of the bottom liquid behind the smooth bore h_2 changes slowly. Therefore the theoretical solution is realized, strictly speaking, at one moment in time and the experimental value of h_2 does not exactly coincide with the theoretical value in all illustrations presented below. But if the experimental value of this depth is used in the calculations, then (2) describes well the wave profile. In calculating all the presented theoretical profiles of smooth bores it is precisely the experimental value h_2 that was employed.

Information about the evolution of a smooth bore in a viscous liquid is contained in Fig. 2, which shows one and the same wave of rising level propagating to the left. Here and in the remaining figures the origin of the coordinates on the x axis is positioned at a point at which $y = (y_1 + y_2)/2$; the lines show the calculation based on the formula (2) and the dots show the experimental data. For a depth and velocity of the incident flow $y_1 = 0.475$ and $F_1 = 0.154$, we obtain for the depth of the bottom liquid after passage of the wave and its velocity from (1) $y_2 = 0.685$ and $V = -0.153$. In the experiment the velocity of propagation was the same as the depth of the bottom liquid 0.683 and 0.671 at a distance $x_0/H = 13.3$ (the curve 1 and the black-colored points) and $(x_0 + \Delta x)/H = 17.5$ (the curve 2 and the light-colored points), respectively.

Figure 3 shows two smooth bores of rising level in the presence of an initial velocity shear between the layers. The probe from which the experimental data were obtained lies at a distance $x_0/H = 13.3$ from the wave generator. Curve 2 and the black points were obtained for $y_1 = 0.475$ and $F_1 = 0.154$, the theory gives $y_2 = 0.685$ and $V = -0.153$, and experiment gives 0.683 and -0.153, respectively; the curve 1 and the light-colored points were obtained for $y_1 = 0.583$ and $F_1 = 0.188$, the theory gives $y_2 = 0.729$ and $V = -0.138$, while experiment gives 0.727 and -0.137.

Figure 4 compares the experimental and computed parameters of the incident and reflected waves. The tracing was obtained from the same probe, lying at a distance $x_0/H = 12$ from the barrier. The velocity of propagation of the wave in the experiment was calculated from the time required for it to traverse the distance from the probe to the wall and back. Curve 2 and the black points refer to the incident wave, propagating to the left, and curve 1 and the light-colored points refer to the reflected wave, moving to the right. Here $y_1 = 0.280$; the theory gives $y_2 = 0.528$, $y_2^! = 0.776$, $|V| = 0.236$, and experiment gives 0.522; 0.756; 0.230, respectively.

The amplitude of the smooth bores generated in the experiments cannot be too high, since they generate a velocity shear between the layers which increases as the amplitude of the wave increases. In the absence of surface tension any shear flow is unstable. Under the conditions of the experiments the surface tension at the interface (σ = 34 dynes/cm) suppressed the appearance of the Kelvin–Helmholtz instability to a velocity difference of 19 cm/sec [8]. From [1] we obtain in dimensionless form for the velocity shear arising between the layers

$$\Delta u = \frac{\sqrt{\mu}}{(1+\sqrt{\lambda})}\frac{(y_2 - y_1)}{y_2(1 - y_2)}\sqrt{gH}. \tag{3}$$

For a sufficiently large amplitude of the smooth bore the velocity shear behind it will be greater than the critical shear and an instability will develop.

Figure 5 shows the tracing of a smooth bore, recorded on an automatic plotter, of rising level, propagating to the left, for $\lambda = 0.8$, H = 6 cm. At the same time, according to (3), the velocity shear which appears Δu = 23.3 cm/sec is greater than its limiting value. Short-wavelength disturbances, characteristic for the Kelvin–Helmholtz instability, are visible behind the smooth bore.

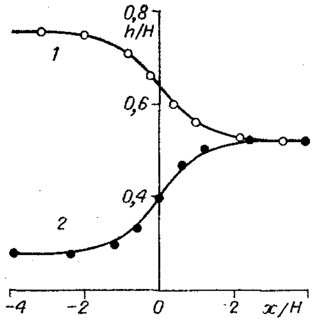

Fig. 4

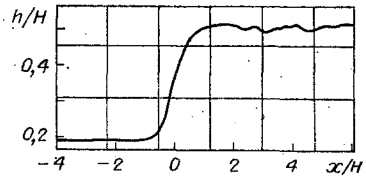

Fig. 5

In conclusion we point out that in hydrodynamics it is rarely possible to obtain analytical solutions and to realize simultaneously the physical process agreeing with these solutions with the accuracy illustrated by the experimental data presented above.

I thank L. V. Ovsyannikov and V. I. Bukreev for his initiative in providing the experimental data and a useful discussion of the results obtained.

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